ELECTRON TRANSITIONS ON DEEP DIRAC LEVELS I

JAROMIR A. MALY *Applied Science Consultants*
5819 Ettersberg Drive, San Jose, California 95123

JAROSLAV VÁVRA *Applied Science Consultants*
67 Pine Lane, Los Altos, California 94022

The original solutions of the Schrödinger relativistic equation and the Dirac equation for hydrogen-like atoms were analyzed for the possible existence of some other electron levels, which were not originally derived. It was found that besides the known atomic levels, each atom should also have the deep Dirac levels (DDLs). The electron transition on such DDLs would produce large amounts of atomic energy (400 to 510 keV per transition depending on the Z of the atom). A possible explanation is given for the excess heat effect observed recently in the electrolysis of lithium or potassium ions, based on existing Dirac quantum theory. The same calculation technique is applied to atoms formed from elementary particles such as e^-e^+, $\mu^+\mu^-$, $\tau^+\tau^-$, $e^-\mu^+$, $e^-\tau^+$, $\mu^-\tau^+$, etc.

I. INTRODUCTION

Quantum theory was established by Bohr in 1913 and developed by Sommerfeld in 1916. It was used to explain the line spectra of hydrogen and other one-electron atoms. Nonrelativistic quantum mechanics was developed from the original deBroglie hypothesis (1924) of a wave connected with the moving particle, in the form of the wave equation by Schrödinger in 1926. Heisenberg added the uncertainty principle to this theory in 1927. Relativistic quantum mechanics was formulated in 1928 by Dirac.

Since the origin of quantum mechanics in 1926 and Dirac quantum mechanics in 1928, only the well-known experimental spectral lines of hydrogen, helium, lithium, sodium, potassium, etc. spectra were explained. There has been no attempt to search theoretically or experimentally for some other energy levels that could produce unknown spectral lines, which still could exist. This technical note describes the search for other energy levels using the known formulas of relativistic Schrödinger quantum mechanics and Dirac quantum mechanics. We also discuss various possible experimental searches to verify our theory.

The same calculation technique is applied for the calculation of atomic levels in the hydrogen-like atom of positron

with negative electron (positronium) and other atoms formed from other elementary particles.

This technical note also gives an explanation for the excess heat effect observed recently in the electrolysis of lithium or potassium ions, based on existing Dirac quantum theory.

II. NEGLECTED SOLUTIONS OF THE RELATIVISTIC SCHRÖDINGER EQUATION FOR HYDROGEN-LIKE ATOMS

It is well known in textbooks of quantum mechanics that the relativistic Schrödinger equation for a hydrogen-like atom can be formulated [see Ref. 1, Eq. (51.12)]:

$$(E - e\phi)^2 u = [-\hbar^2 c^2 \nabla^2 + 2iehcA \cdot \nabla$$
$$+ iehc(\nabla \cdot A) + e^2 A^2 + m^2 c^4] u \quad (1)$$

with $\psi(r,t) = u(r)\exp(-iEt/\hbar)$, where

$\phi(r,t), A(r,t)$ = electromagnetic potentials

m, e = mass and charge of electron

E = energy of electron

c = velocity of light

t = time.

For the case of $A = 0$ and $\phi(r)$ being spherically symmetric, Eq. (1) can be simplified to [see Ref. 1, Eq. (51.13)]:

$$(-\hbar c^2 \nabla^2 + m^2 c^4) u(r) = [E - e\phi(r)]^2 u(r) . \quad (2)$$

This equation can be separated in spherical coordinates (see Ref. 1),

$$u(r, \theta, \phi) = R(r) Y_{lm}(\theta, \phi) , \quad (3)$$

and this yields the radial equation [see Ref. 1, Eq. (51.15)]

$$\frac{1}{\rho^2} \frac{d}{d\rho} \left(\rho^2 \frac{dR}{d\rho} \right) + \left[\frac{\lambda}{\rho} - \frac{1}{4} - \frac{l(l+1) - \gamma^2}{\rho^2} \right] R = 0 \quad (4)$$

with

$$\rho = \alpha r , \quad \gamma = \frac{Ze^2}{\hbar c} , \quad e\phi = -Ze^2/r ,$$

$$\alpha^2 = \frac{4(m^2 c^4 - E^2)}{\hbar^2 c^2} , \quad \lambda = \frac{2E\gamma}{\hbar c\alpha} . \quad (5)$$

From Eq. (5) we can express the formula for energy levels [see Ref. 1, Eq. (51.16)]:

$$E = mc^2 \left(1 + \frac{\gamma^2}{\lambda^2}\right)^{-1/2} . \tag{6}$$

Equation (6) was first derived by Sommerfeld (1916) on the basis of old quantum theory. Equation (6) contains λ, which can be obtained by the solution of Eq. (4). This solution is obtained by substitution:

$$R(\rho) = F(\rho)\exp(-\rho/2) , \tag{7}$$

where $F(\rho)$ is a polynomial of finite order that can be expressed in the form [see Ref. 1, Eq. (16.11)]

$$F(\rho) = \rho^s(a_0 + a_1\rho + a_2\rho^2 + \ldots) = \rho^s L(\rho) . \tag{8}$$

Introducing Eq. (8) into Eq. (4), we have the differential equation

$$\rho^2 L'' + \rho[2(s + 1) - \rho]L'$$
$$+ [\rho(\lambda - s - 1) + s(s + 1) - l(l + 1) + \gamma^2]L = 0 . \tag{9}$$

This equation for $\rho = 0$ is zero, only if the following is satisfied:

$$s(s + 1) + \gamma^2 - l(l + 1) = 0 ,$$

and it gives the solution [see Ref. 1, Eq. (51.19)]

$$s = -\tfrac{1}{2} \pm [(l + \tfrac{1}{2})^2 - \gamma^2]^{1/2} . \tag{10}$$

For $l = 0$, both solutions for s in Eq. (10) are negative, with plus or minus sign before the square root.

We can also calculate the values of coefficients a_0, a_1, $a_2, \ldots, a_{n'}$ in the polynomial expansion of Eq. (8) by the following recurrent formula:

$$-a_{n'+1} = a_{n'}[K2 - n']/[(n' + 1)(n' + K1)] \tag{11}$$

with the notation

$$K1 = 2\{\tfrac{1}{2} \pm [(l + \tfrac{1}{2})^2 - \gamma^2]^{1/2}\} = 2(s + 1)$$

and

$$K2 = \lambda - \frac{K1}{2} .$$

Further it is found[1] that to keep the solution finite at infinite r, the $L(\rho)$ series must terminate. This termination is achieved by putting

$$\lambda = n' + s + 1 , \tag{12}$$

which results in $a_{n'+1} = 0$.

This method of solution shows that only integer radial numbers are possible, to terminate the series. As a result, in accepted quantum mechanics, no fractional quantum numbers are possible.

Equation (10), which defines the energy levels, contains the variable s. Since the conception of quantum mechanics, only the solution with the plus sign in variable s was used [see Eq. (10)]. This choice was preferred because at $r = \rho = 0$, the solution $F(\rho)$ with negative s has infinity, similarly as the potential $V = Ze^2/r$ has the infinite value at $r = 0$.

The choice of the positive sign in the s was justified on the basis that γ^2 is a small value and the solution at $r = 0$ is only slightly negative for $l = 0$. Using this choice, the energy levels were calculated and used until now as the only possible energy levels of hydrogen-like atoms.

However, this basic approach may be incorrect. We cannot eliminate the solutions with the negative sign inside the s parameter. The reason is simple: The value of γ^2 for larger atoms is not small (for example, for the uranium ion with one single electron, it has a value of 0.45). Therefore, the s always has a negative value for $l = 0$. It sometimes has a value of -0.5 even with the positive sign inside s. As a result, the solution $F(\rho)$ has infinity at $\rho = 0$ for both negative and positive signs inside s.

The fact that $F(\rho)$ has infinite value at $r = 0$ is not important because the entire solution is valid only between the rim of the nucleus and infinity. The Coulomb potential inside the nucleus is not Ze^2/r, but it is finite; for example, for the uniformly distributed charge, it is (R is the radius of the nucleus) as follows[2]:

$$V(r) = -\left[\frac{3}{2} - \frac{1}{2}\left(\frac{r^2}{R^2}\right)\right]\frac{Ze^2}{R} . \tag{13}$$

This potential is finite inside the nucleus (at $r = 0$, its value is $-3Ze^2/2R$), and the solution of the relativistic Schrödinger equation inside the nucleus is completely different from the solution $F(\rho)$ mentioned earlier. This solution has to be matched on the surface of the nucleus with the external solution Eqs. (8) through (12) (to have the same values on the nuclear surface). After matching both solutions, the wave function of the electron can be fully normalized in the entire region $0 < r < \infty$.

For our discussion, however, it is only necessary that levels with both plus or minus signs in the parameter s are valid solutions. This should be verified in further experimental work.

III. NEGLECTED SOLUTIONS OF THE DIRAC EQUATION FOR HYDROGEN-LIKE ATOMS

For the computation of energy levels, we need be concerned only with the radial part of the solution of the Dirac equation, Eq. (1). It is shown in advanced quantum mechanics (problem 200 in Ref. 3) that the Dirac equation for hydrogen-like atoms can be separated into spherical harmonic functions multiplied by radial functions F/r or G/r.

The differential equations for radial functions F and G are shown in the following [see Ref. 1, Eqs. (53.16) and (53.17)]:

$$\left(\frac{d}{d\rho} + \frac{k}{\rho}\right)G - \left(\frac{\alpha_2}{\alpha} + \frac{V}{\hbar c \alpha}\right)F = 0$$

and

$$\left(\frac{d}{d\rho} + \frac{k}{\rho}\right)F - \left(\frac{\alpha_1}{\alpha} - \frac{V}{\hbar c \alpha}\right)G = 0 \tag{14}$$

with

$$\alpha_1 = \frac{(mc^2 + E)}{\hbar c} , \quad \alpha_2 = \frac{(mc^2 - E)}{\hbar c} ,$$

$$\rho = \alpha r , \quad \alpha = (\alpha_1 \alpha_2)^{1/2} = (m^2 c^4 - E^2)^{1/2}/\hbar c , \tag{15}$$

where

$$V(r) = -Ze^2/r = \text{Coulomb potential outside the nucleus}$$

$$m, e = \text{mass and charge of electron}$$

E = energy of electron

c = velocity of light.

Further, we introduce the abbreviation and express F and G functions in the form

$$\gamma = \frac{Ze^2}{\hbar c} \ , \quad \frac{V}{\hbar c \alpha} = -\frac{\gamma}{\rho} \ ,$$

$$F(\rho) = f(\rho)\exp(-\rho) \ , \quad G(\rho) = g(\rho)\exp(-\rho) \ .$$

For the case when $V(r)$ is spherically symmetric, Eq. (14) can be simplified to the following [see Ref. 1, Eq. (53.18)]:

$$g' - g + \frac{kg}{\rho} - \left(\frac{\alpha_2}{\alpha} - \frac{\gamma}{\rho}\right)f = 0$$

and

$$f' - f + \frac{kf}{\rho} - \left(\frac{\alpha_1}{\alpha} - \frac{\gamma}{\rho}\right)g = 0 \ . \tag{16}$$

The solution of f and g can be searched in the polynomial form of finite order, which can be expressed as follows [see Ref. 1, Eq. (53.20)]:

$$f(\rho) = \rho^s(a_0 + a_1\rho + a_2\rho^2 + \ldots) \ , \quad a_0 \neq 0 \ ,$$

and

$$g(\rho) = \rho^s(b_0 + b_1\rho + b_2\rho^2 + \ldots) \ , \quad b_0 \neq 0 \ . \tag{17}$$

Introducing Eq. (17) into Eq. (16) and equating the coefficients at $\rho^{s+\nu-1}$ to zero gives the recurrent formula equations for $\nu > 0$ [see Ref. 1, Eq. (53.21)]:

$$(s + \nu + k)b_\nu - b_{\nu-1} + \gamma a_\nu - \frac{\alpha_1}{\alpha} a_{\nu-1} = 0$$

and

$$(s + \nu - k)a_\nu - a_{\nu-1} - \gamma b_{\nu-1} - \frac{\alpha_1}{\alpha} b_{\nu-1} = 0 \ . \tag{18}$$

These equations for $\nu = 0$ are zero only if the following are satisfied [see Ref. 1, Eq. (53.22)]:

$$(s + k)b_0 + \gamma a_0 = 0$$

and

$$(s - k)a_0 - \gamma b_0 = 0 \ , \tag{19}$$

which has the nonvanishing solution for a_0 and b_0 only if the determinant of Eq. (19) vanishes. This leads to the following solution [see Ref. 1, Eq. (53.23)]:

$$s = \pm(k^2 - \gamma^2)^{1/2} \ . \tag{20}$$

Further, we can calculate the values of $a_0, a_1, a_2, \ldots$ and $b_0, b_1, b_2, \ldots$ from Eq. (18) by the recurrent formula [see Ref. 1, Eq. (53.23)]

$$b_\nu[\alpha(s + \nu + k) + \alpha_2\gamma] = a_\nu[\alpha_2(s + \nu - k) - \alpha\gamma] \ . \tag{21}$$

In Ref. 1, it is found that to keep the solution finite at infinite r, both of the series in Eq. (17) must terminate. This termination is provided by putting $\nu = n'$, so that both Eqs. (18) yield the following [see Ref. 1, Eq. (53.25)]:

$$\alpha_2 a_{n'} = -\alpha b_{n'} \quad \text{and} \quad n' = 0,1,2,3,\ldots \ , \tag{22}$$

with

$$k = 1,2,3\ldots \quad \text{or} \quad -1,-2,-3\ldots$$

$$n = n' + |k| = \text{main quantum number} \ .$$

By solving Eq. (18) with $\nu = n'$ and using Eq. (22), we find the equation

$$2\alpha(s + n') = \gamma(\alpha_1 - \alpha_2) = \frac{2E\gamma}{\hbar c} \ . \tag{23}$$

Inserting α_1, α_2, and α from Eq. (15) and making the square of Eq. (23) gives the equation

$$(m^2c^4 - E^2)(s + n')^2 = E^2\gamma^2 \ ,$$

which is easily solved in the following form [see Ref. 1, Eq. (53.26)]:

$$E = mc^2\left[1 + \frac{\gamma^2}{(s + n')^2}\right]^{-1/2} \ . \tag{24}$$

This method of solution also shows that only integer radial numbers n' are possible, to terminate the series. As a result, in the accepted Dirac quantum mechanics, no fractional quantum numbers are possible.

Equation (24) used for the calculation of the energy levels E contains the s variable. Since the conception of quantum mechanics, only the solution with the plus sign in the definition of s variable was used [see Eq. (20)]. This choice was preferred because at $r = 0$, the solution $F(\rho)$ with negative s has infinity, and similarly, the potential $V = Ze^2/r$ has the infinite value. Using this choice, the energy levels were calculated and used until now as the only possible energy levels of hydrogen-like atoms. This basic approach may, however, be incorrect. We believe that we cannot eliminate the solutions with the negative sign of the s parameter.

The fact that F and G have infinite value at $r = 0$ is not important because the entire calculated solution is valid only between the rim of the nucleus and infinity. The Coulomb potential inside the nucleus is not Ze^2/r, but it is finite. For example, for the uniformly distributed charge, the potential is given in Eq. (13), according to Ref. 2. This type of potential (in the vicinity of $r = 0$ inside the nucleus) is used for complex atoms in the normal Dirac Hartree-Fock (DHF) calculations of electron energy levels with the finite nucleus.[4,5]

We conclude as in Sec. II. that the solution for the electron inside the finite nucleus has to be done separately. Such a solution has to be matched on the surface of the nucleus with the external solution derived in Eqs. (14) through (24), and both solutions can be normalized in the entire space.

Both plus and minus signs of the s parameter give valid solutions of the Dirac equation.

IV. CALCULATIONS OF NEW ENERGY LEVELS

A computer program was written that calculates atomic energy levels according to Eq. (6) [Relativistic Schrödinger levels $E1S(+)$ and $E2S(-)$ in Tables I.A through I.F] and according to Eq. (24) [Dirac levels $ED1(+)$ and $ED2(-)$ in Tables II.A through II.F]. For comparison, we also show the nonrelativistic Schrödinger levels given by a simple Bohr formula: $E(N,Z) = -hcRy(Z/n)^2 = -13.605826(Z/N)^2$, calculated for the mass of the electron $mc^2 = 510\,999.06$ eV and for $hcRy = 13.605826$ eV [see $E(N,Z)$ columns in the tables].

Some results are quoted in Tables I and II for hydrogen, lithium, sodium, potassium, rubidium, and cesium atoms, calculated for the main quantum numbers 1 through 6.

This program calculates the energy levels in electron volts for all possible combinations of quantum numbers $n = N$, $n' = M$ and $l = L$ for the Schrödinger levels, and for quantum numbers $n = N$, $k = K$, $l = L1 = k - 1$ (used with $k > 0$),

TABLE I.A

Relativistic Schrödinger Levels for Hydrogen ($Z = 1$)*

	$E(N,Z)$	N	M	L	E1S	E2S
1s	−13.605826	1	0	0	−13.606597	−507 271.937500
2p	−3.501457	2	0	1	−3.401449	−13.605632[a]
2s	−3.401457	2	1	0	−3.401570	−13.603699
3d	−1.511759	3	0	2	−1.511747	−3.401425[a]
3p	−1.511759	3	1	1	−1.511755	−509 755.250000
3s	−1.511759	3	2	0	−1.511790	−3.401207
4f	−1.511764	4	0	3	−0.850357	−1.511744[a]
4d	−0.850364	4	1	2	−0.850358	−13.605434[a]
4p	−0.850364	4	2	1	−0.850361	−13.604666
4s	−0.850364	4	3	0	−0.850376	−1.511683
5g	−0.850364	5	0	4	−0.544228	−0.850356[a]
5f	−0.544233	5	1	3	−0.544228	−3.401415[a]
5d	−0.544233	5	2	2	−0.544229	−510 264.468750
5p	−0.544233	5	3	1	−0.544231	−3.401328
5s	−0.544233	5	4	0	−0.544238	−0.850331
6h	−0.544233	6	0	5	−0.377936	−0.544228[a]
6g	−0.377940	6	1	4	−0.377936	−1.511743[a]
6f	−0.377940	6	2	3	−0.377936	−13.605356[a]
6d	−0.377940	6	3	2	−0.377937	−13.604863
6p	−0.377940	6	4	1	−0.377938	−1.511719
6s	−0.377940	6	5	0	−0.377942	−0.544215

*In electron volts.
[a]Negative energy states, not observable.

TABLE I.B

Relativistic Schrödinger Levels for Lithium ($Z = 3$)*

	$E(N,Z)$	N	M	L	E1S	E2S
1s	−122.452431	1	0	0	−122.524666	−499 808.906250
2p	−30.613108	2	0	1	−30.614946	−122.446335[a]
2s	−30.613108	2	1	0	−30.624737	−122.289948
3d	−13.605826	3	0	2	−13.606018	−30.612989[a]
3p	−13.605826	3	1	1	−13.606598	−507 270.531250
3s	−13.605826	3	2	0	−13.609498	−30.595381
4f	−7.653277	4	0	3	−7.653292	−13.605768[a]
4d	−7.653277	4	1	2	−7.653397	−122.430687[a]
4p	−7.653277	4	2	1	−7.653641	−122.368149
4s	−7.653277	4	3	0	−7.654865	−13.600801
5g	−4.898098	5	0	4	−4.898082	−7.653233[a]
5f	−4.898098	5	1	3	−4.898112	−30.612150[a]
5d	−4.898098	5	2	2	−4.898166	−508 761.906250
5p	−4.898098	5	3	1	−4.898292	−30.605167
5s	−4.898098	5	4	0	−4.898918	−7.651195
6h	−3.401457	6	0	5	−3.401438	−4.898064[a]
6g	−3.401457	6	1	4	−3.401449	−13.605629[a]
6f	−3.401457	6	2	3	−3.401466	−122.423981[a]
6d	−3.401457	6	3	2	−3.401498	−122.383781
6p	−3.401457	6	4	1	−3.401570	−13.603700
6s	−3.401457	6	5	0	−3.401933	−4.897039

*In electron volts.
[a]Negative energy states, not observable.

TABLE I.C

Relativistic Schrödinger Levels for Sodium $(Z = 11)$*

	$E(N,Z)$	N	M	L	$E1S$	$E2S$
$1s$	−1646.304932	1	0	0	−1659.730713	−469 847.125000
$2p$	−411.576233	2	0	1	−411.959534	−1 645.406860[a]
$2s$	−411.576233	2	1	0	−413.751404	−1 617.428955
$3d$	−182.922775	3	0	2	−182.979919	−411.605316[a]
$3p$	−182.922775	3	1	1	−183.084915	−497 321.375000
$3s$	−182.922775	3	2	0	−183.615417	−408.425690
$4f$	−102.894058	4	0	3	−102.909325	−182.934998[a]
$4d$	−102.894058	4	1	2	−102.928276	−1 642.586548[a]
$4p$	−102.894058	4	2	1	−102.972580	−1 631.388794
$4s$	−102.894058	4	3	0	−103.196243	−182.035324
$5g$	−65.852203	5	0	4	−65.857674	−102.898788[a]
$5f$	−65.852203	5	1	3	−65.863060	−411.453827[a]
$5d$	−65.852203	5	2	2	−65.872772	−502 795.125000
$5p$	−65.852203	5	3	1	−65.895447	−410.194550
$5s$	−65.852203	5	4	0	−66.009918	−102.529335
$6h$	−45.730694	6	0	5	−45.733036	−65.854256[a]
$6g$	−45.730694	6	1	4	−45.735012	−182.910019[a]
$6f$	−45.730694	6	2	3	−45.738132	−1 641.381470[a]
$6d$	−45.730694	6	3	2	−45.743752	−1 634.180420
$6p$	−45.730694	6	4	1	−45.756870	−182.561264
$6s$	−45.730694	6	5	0	−45.823097	−65.668388

*In electron volts.
[a]Negative energy states, not observable.

TABLE I.D

Relativistic Schrödinger Levels for Potassium $(Z = 19)$*

	$E(N,Z)$	N	M	L	$E1S$	$E2S$
$1s$	−4911.703125	1	0	0	−5034.691895	−439 444.250000
$2p$	−1227.925781	2	0	1	−1231.371826	−4 903.845215[a]
$2s$	−1227.925781	2	1	0	−1247.764404	−4 660.028320
$3d$	−545.744873	3	0	2	−546.264771	−1 228.209595[a]
$3p$	−545.744873	3	1	1	−547.203064	−487 357.031250
$3s$	−545.744873	3	2	0	−552.048096	−1 199.946045
$4f$	−306.981445	4	0	3	−307.123444	−545.864441[a]
$4d$	−306.981445	4	1	2	−307.292419	−4 878.896484[a]
$4p$	−306.981445	4	2	1	−307.688324	−4 781.223633
$4s$	−306.981445	4	3	0	−309.728546	−537.824463
$5g$	−196.468124	5	0	4	−196.520752	−307.029633[a]
$5f$	−196.468124	5	1	3	−196.568771	−1 226.861572[a]
$5d$	−196.468124	5	2	2	−196.655334	−496 824.031250
$5p$	−196.468124	5	3	1	−196.858002	−1 215.713867
$5s$	−196.468124	5	4	0	−197.901199	−303.719696
$6h$	−136.436218	6	0	5	−136.459702	−196.490250[a]
$6g$	−136.436218	6	1	4	−136.477371	−545.642273[a]
$6f$	−136.436218	6	2	3	−136.505157	−4 868.289063[a]
$6d$	−136.436218	6	3	2	−136.555252	−4 805.430176
$6p$	−136.436218	6	4	1	−136.672516	−542.545166
$6s$	−136.436218	6	5	0	−137.275650	−194.822815

*In electron volts.
[a]Negative energy states, not observable.

TABLE I.E
Relativistic Schrödinger Levels for Rubidium ($Z = 37$)*

	$E(N,Z)$	N	M	L	$E1S$	$E2S$
$1s$	$-18\,626.375000$	1	0	0	$-20\,644.611328$	$-367\,219.500000$
$2p$	$-4\,656.593750$	2	0	1	$-4\,706.923340$	$-18\,516.222656^a$
$2s$	$-4\,656.593750$	2	1	0	$-4\,974.706543$	$-15\,279.839844$
$3d$	$-2\,069.597168$	3	0	2	$-2\,077.166504$	$-4\,660.842285^a$
$3p$	$-2\,069.597168$	3	1	1	$-2\,090.895752$	$-464\,820.000000$
$3s$	$-2\,069.597168$	3	2	0	$-2\,169.395264$	$-4\,254.942871$
$4f$	$-1\,164.148438$	4	0	3	$-1\,166.227295$	$-2\,071.379639^a$
$4d$	$-1\,164.148438$	4	1	2	$-1\,168.674072$	$-18\,166.330078^a$
$4p$	$-1\,164.148438$	4	2	1	$-1\,174.468506$	$-16\,871.996094$
$4s$	$-1\,164.148438$	4	3	0	$-1\,207.331421$	$-1\,953.266113$
$5g$	-745.055054	5	0	4	-745.833618	$-1\,164.875000^a$
$5f$	-745.055054	5	1	3	-746.526611	$-4\,641.485352^a$
$5d$	-745.055054	5	2	2	-747.780090	$-483\,364.562500$
$5p$	-745.055054	5	3	1	-750.745605	$-4\,484.783691$
$5s$	-745.055054	5	4	0	-767.481384	$-1\,115.723389$
$6h$	-517.399292	6	0	5	-517.751831	-745.393738^a
$6g$	-517.399292	6	1	4	-518.006531	$-2\,068.184082^a$
$6f$	-517.399292	6	2	3	-518.407776	$-18\,020.744141^a$
$6d$	-517.399292	6	3	2	-519.133240	$-17\,185.626953$
$6p$	-517.399292	6	4	1	-520.848267	$-2\,024.092896$
$6s$	-517.399292	6	5	0	-530.495850	-720.476013

*In electron volts.
[a]Negative energy states, not observable.

TABLE I.F
Relativistic Schrödinger Levels for Cesium ($Z = 55$)*

	$E(N,Z)$	N	M	L	$E1S$	$E2S$
$1s$	$-41\,157.6250$	1	0	0	$-54\,466.6953$	$-281\,441.9063$
$2p$	$-10\,289.4062$	2	0	1	$-10\,540.7666$	$-40\,639.3320^a$
$2s$	$-10\,289.4062$	2	1	0	$-12\,271.9512$	$-26\,313.8281$
$3d$	$-4\,573.0698$	3	0	2	$-4\,610.3994$	$-10\,310.6855^a$
$3p$	$-4\,573.0698$	3	1	1	$-4\,679.3872$	$-442\,007.7187$
$3s$	$-4\,573.0698$	3	2	0	$-5\,176.7700$	$-8\,283.7519$
$4f$	$-2\,572.3516$	4	0	3	$-2\,582.6133$	$-4\,581.8906^a$
$4d$	$-2\,572.3516$	4	1	2	$-2\,594.6746$	$-38\,996.6914^a$
$4p$	$-2\,572.3516$	4	2	1	$-2\,623.8232$	$-33\,411.8125$
$4s$	$-2\,572.3516$	4	3	0	$-2\,829.2942$	$-3\,968.0662$
$5g$	$-1\,646.3049$	5	0	4	$-1\,650.1349$	$-2\,575.9734^a$
$5f$	$-1\,646.3049$	5	1	3	$-1\,653.5461$	$-10\,216.3867^a$
$5d$	$-1\,646.3049$	5	2	2	$-1\,659.7292$	$-469\,851.7500$
$5p$	$-1\,646.3049$	5	3	1	$-1\,674.6537$	$-9\,478.5058$
$5s$	$-1\,646.3049$	5	4	0	$-1\,778.3944$	$-2\,315.3423$
$6h$	$-1\,143.2675$	6	0	5	$-1\,145.0157$	$-1\,648.0028^a$
$6g$	$-1\,143.2675$	6	1	4	$-1\,146.2645$	$-4\,566.2959^a$
$6f$	$-1\,143.2675$	6	2	3	$-1\,148.2139$	$-38\,336.0508^a$
$6d$	$-1\,143.2675$	6	3	2	$-1\,151.8384$	$-34\,723.2813$
$6p$	$-1\,143.2675$	6	4	1	$-1\,160.4276$	$-4\,354.4897$
$6s$	$-1\,143.2675$	6	5	0	$-1\,219.8516$	$-1\,514.2912$

*In electron volts.
[a]Negative energy states, not observable.

TABLE II.A

Dirac Levels of Hydrogen-Like Atoms for Hydrogen $(Z = 1)$*

	$E(N,Z)$	N	M	K	$L1$	E1D	$L2$	E2D
$1s$	−13.605826	1	0	1	0	−13.605873	1	−13.605873[a]
$2p$	−3.401457	2	0	2	1	−3.401434	2	−3.401434[a]
$2s$	−3.401457	2	1	1	0	−3.401479	1	−509 133.375000
$3d$	−1.511759	3	0	3	2	−1.511746	3	−1.511746[a]
$3p$	−1.511759	3	1	2	1	−1.511750	2	−13.605512[a]
$3s$	−1.511759	3	2	1	0	−1.511764	1	−13.604422
$4f$	−0.850364	4	0	4	3	−0.850356	4	−0.850356[a]
$4d$	−0.850364	4	1	3	2	−0.850357	3	−3.401419[a]
$4p$	−0.850364	4	2	2	1	−0.850359	2	−510 064.125000
$4s$	−0.850364	4	3	1	0	−0.850365	1	−3.401298
$5g$	−0.544233	5	0	5	4	−0.544228	5	−0.544228[a]
$5f$	−0.544233	5	1	4	3	−0.544228	4	−1.511744[a]
$5d$	−0.544233	5	2	3	2	−0.544229	3	−13.605389[a]
$5p$	−0.544233	5	3	2	1	−0.544230	2	−13.604785
$5s$	−0.544233	5	4	1	0	−0.544233	1	−1.511710
$6h$	−0.377940	6	0	6	5	−0.377936	6	−0.377936[a]
$6g$	−0.377940	6	1	5	4	−0.377936	5	−0.850356[a]
$6f$	−0.377940	6	2	4	3	−0.377936	4	−3.401412[a]
$6d$	−0.377940	6	3	3	2	−0.377937	3	−510 381.343750
$6p$	−0.377940	6	4	2	1	−0.377937	2	−3.401344
$6s$	−0.377940	6	5	1	0	−0.377939	1	−0.850342

*In electron volts.
[a]Negative energy states, not observable.

TABLE II.B

Dirac Levels of Hydrogen-Like Atoms for Lithium $(Z = 3)$*

	$E(N,Z)$	N	M	K	$L1$	E1D	$L2$	E2D
$1s$	−122.452431	1	0	1	0	−122.465904	1	−122.465904[a]
$2p$	−30.613108	2	0	2	1	−30.613724	2	−30.613724[a]
$2s$	−30.613108	2	1	1	0	−30.617392	1	−505 405.062500
$3d$	−13.605826	3	0	3	2	−13.605872	3	−13.605872[a]
$3p$	−13.605826	3	1	2	1	−13.606235	2	−122.436554[a]
$3s$	−13.605826	3	2	1	0	−13.607322	1	−122.348587
$4f$	−7.653277	4	0	4	3	−7.653259	4	−7.653259[a]
$4d$	−7.653277	4	1	3	2	−7.653364	3	−30.612501[a]
$4p$	−7.653277	4	2	2	1	−7.653488	2	−508 202.625000
$4s$	−7.653277	4	3	1	0	−7.653947	1	−30.602724
$5g$	−4.898098	5	0	5	4	−4.898073	5	−4.898073[a]
$5f$	−4.898098	5	1	4	3	−4.898095	4	−13.605691[a]
$5d$	−4.898098	5	2	3	2	−4.898135	3	−122.426781[a]
$5p$	−4.898098	5	3	2	1	−4.898213	2	−122.377914
$5s$	−4.898098	5	4	1	0	−4.898448	1	−13.602975
$6g$	−3.401457	6	0	6	5	−3.401434	6	−3.401434[a]
$6h$	−3.401457	6	1	5	4	−3.401443	5	−7.653214[a]
$6f$	−3.401457	6	2	4	3	−3.401456	4	−30.611889[a]
$6d$	−3.401457	6	3	3	2	−3.401479	3	−509 134.750000
$6p$	−3.401457	6	4	2	1	−3.401525	2	−30.606386
$6s$	−3.401457	6	5	1	0	−3.401661	1	−7.652112

*In electron volts.
[a]Negative energy states, not observable.

TABLE II.C

Dirac Levels of Hydrogen-Like Atoms for Sodium ($Z = 11$)*

	$E(N,Z)$	N	M	K	$L1$	$E1D$	$L2$	$E2D$
1s	−1646.304932	1	0	1	0	−1648.949341	1	−1 648.949341[a]
2p	−411.576233	2	0	2	1	−411.738037	2	−411.738037[a]
2s	−411.576233	2	1	1	0	−412.403717	1	−490 473.187500
3d	−182.922775	3	0	3	2	−182.953720	3	−182.953720[a]
3p	−182.922775	3	1	2	1	−183.019272	2	−1 643.643066[a]
3s	−182.922775	3	2	1	0	−183.216553	1	−1 627.902588
4f	−102.894058	4	0	4	3	−102.903412	4	−102.903412[a]
4d	−102.894058	4	1	3	2	−102.917229	3	−411.516968[a]
4p	−102.894058	4	2	2	1	−102.944885	2	−500 742.625000
4s	−102.894058	4	3	1	0	−103.028099	1	−409.753845
5g	−65.852203	5	0	5	4	−65.855789	5	−65.855789[a]
5f	−65.852203	5	1	4	3	−65.860031	4	−182.920990[a]
5d	−65.852203	5	2	3	2	−65.867104	3	−1 641.883667[a]
5p	−65.852203	5	3	2	1	−65.881264	2	−1 633.132935
5s	−65.852203	5	4	1	0	−65.923866	1	−182.430344
6h	−45.730694	6	0	6	5	−45.732288	6	−45.732288[a]
6g	−45.730694	6	1	5	4	−45.733921	5	−102.895111[a]
6f	−45.730694	6	2	4	3	−45.736378	4	−411.406586[a]
6d	−45.730694	6	3	3	2	−45.740475	3	−504 162.187500
6p	−45.730694	6	4	2	1	−45.748669	2	−410.414795
6s	−45.730694	6	5	1	0	−45.773319	1	−102.696281

*In electron volts.
[a]Negative energy states, not observable.

TABLE II.D

Dirac Levels of Hydrogen-Like Atoms for Potassium ($Z = 19$)*

	$E(N,Z)$	N	M	K	$L1$	$E1D$	$L2$	$E2D$
1s	−4911.703125	1	0	1	0	−4935.489746	1	−4 935.489746[a]
2p	−1227.925781	2	0	2	1	−1229.392578	2	−1 229.392578[a]
2s	−1227.925781	2	1	1	0	−1235.365601	1	−475 488.250000
3d	−545.744873	3	0	3	2	−546.031189	3	−546.031189[a]
3p	−545.744873	3	1	2	1	−546.615967	2	−4 888.219238[a]
3s	−545.744873	3	2	1	0	−548.386475	1	−4 751.041992
4f	−306.981445	4	0	4	3	−307.070709	4	−307.070709[a]
4d	−306.981445	4	1	3	2	−307.193848	3	−1 227.422974[a]
4p	−306.981445	4	2	2	1	−307.440643	2	−493 275.781250
4s	−306.981445	4	3	1	0	−308.187256	1	−1 211.814819
5g	−196.468124	5	0	5	4	−195.503967	5	−196.503967[a]
5f	−196.468124	5	1	4	3	−196.541779	4	−545.739502[a]
5d	−196.468124	5	2	3	2	−196.604843	3	−4 872.705078[a]
5p	−196.468124	5	3	2	1	−196.731201	2	−4 796.342285
5s	−196.468124	5	4	1	0	−197.113312	1	−541.380737
6h	−136.436218	6	0	6	5	−136.453064	6	−136.453064[a]
6g	−136.436218	6	1	5	4	−136.467651	5	−306.996857[a]
6f	−136.436218	6	2	4	3	−136.489532	4	−1 226.441162[a]
6d	−136.436218	6	3	3	2	−136.526031	3	−499 187.187500
6p	−136.436218	6	4	2	1	−136.599152	2	−1 217.660278
6s	−136.436218	6	5	1	0	−136.820190	1	−305.227631

*In electron volts.
[a]Negative energy states, not observable.

TABLE II.E

Dirac Levels of Hydrogen-Like Atoms for Rubidium ($Z = 37$)*

	$E(N,Z)$	N	M	K	$L1$	$E1D$	$L2$	$E2D$
$1s$	−18 626.375000	1	0	1	0	−18 978.628906	1	−18 978.628906[a]
$2p$	−4 656.593750	2	0	2	1	−4 677.960449	2	−4 677.960449[a]
$2s$	−4 656.593750	2	1	1	0	−4 766.891113	1	−441 364.062500
$3d$	−2 069.597168	3	0	3	2	−2 073.784912	3	−2 073.784912[a]
$3p$	−2 069.597168	3	1	2	1	−2 082.274170	2	−18 295.759766[a]
$3s$	−2 069.597168	3	2	1	0	−2 108.662842	1	−16 482.818359
$4f$	−1 164.148438	4	0	4	3	−1 165.466187	4	−1 165.466187[a]
$4d$	−1 164.148438	4	1	3	2	−1 167.244507	3	−4 649.525391[a]
$4p$	−1 164.148438	4	2	2	1	−1 170.831421	2	−476 427.093750
$4s$	−1 164.148438	4	3	1	0	−1 181.946533	1	−4 430.021973
$5g$	−745.055054	5	0	5	4	−745.591675	5	−745.591675[a]
$5f$	−745.055054	5	1	4	3	−746.136536	4	−2 069.580566[a]
$5d$	−745.055054	5	2	3	2	−747.047852	3	−18 081.117188[a]
$5p$	−745.055054	5	3	2	1	−748.884827	2	−17 067.585938
$5s$	−745.055054	5	4	1	0	−754.566162	1	−2 007.379150
$6h$	−517.399292	6	0	6	5	−517.656433	6	−517.656433[a]
$6g$	−517.399292	6	1	5	4	−517.866455	5	−1 164.403076[a]
$6f$	−517.399292	6	2	4	3	−518.181946	4	−4 635.475586[a]
$6d$	−517.399292	6	3	3	2	−518.709473	3	−487 980.656250
$6p$	−517.399292	6	4	2	1	−519.772339	2	−4 511.977051
$6s$	−517.399292	6	5	1	0	−523.055420	1	−1 138.996704

*In electron volts.
[a]Negative energy states, not observable.

TABLE II.F

Dirac Levels of Hydrogen-Like Atoms for Cesium ($Z = 55$)*

	$E(N,Z)$	N	M	K	$L1$	$E1D$	$L2$	$E2D$
$1s$	−41 157.625000	1	0	1	0	−42 963.332031	1	−42 963.332031[a]
$2p$	−10 289.406250	2	0	2	1	−10 395.035156	2	−10 395.035156[a]
$2s$	−10 289.406250	2	1	1	0	−10 856.153320	1	−406 227.375000
$3d$	−4 573.069824	3	0	3	2	−4 593.671875	3	−4 593.671875[a]
$3p$	−4 573.069824	3	1	2	1	−4 635.774414	2	−39 594.203125[a]
$3s$	−4 573.069824	3	2	1	0	−4 772.830078	1	−31 787.910156
$4f$	−2 572.351563	4	0	4	3	−2 578.833496	4	−2 578.833496[a]
$4d$	−2 572.351563	4	1	3	2	−2 587.578369	3	−10 255.312500[a]
$4p$	−2 572.351563	4	2	2	1	−2 605.400879	2	−459 463.343750
$4s$	−2 572.351563	4	3	1	0	−2 663.008057	1	−9 220.168945
$5g$	−1 646.304932	5	0	5	4	−1 648.949341	5	−1 648.949341[a]
$5f$	−1 646.304932	5	1	4	3	−1 651.620605	4	−4 573.064453[a]
$5d$	−1 646.304932	5	2	3	2	−1 656.106689	3	−38 608.199219[a]
$5p$	−1 646.304932	5	3	2	1	−1 665.235718	2	−34 227.964844
$5s$	−1 646.304932	5	4	1	0	−1 694.615967	1	−4 272.888672
$6h$	−1 143.267456	6	0	6	5	−1 144.537720	6	−1 144.537720[a]
$6g$	−1 143.267456	6	1	5	4	−1 145.565918	5	−2 573.629883[a]
$6f$	−1 143.267456	6	2	4	3	−1 147.113525	4	−10 187.255859[a]
$6d$	−1 143.267456	6	3	3	2	−1 149.711304	3	−476 740.093750
$6p$	−1 143.267456	6	4	2	1	−1 154.992065	2	−9 605.337891
$6s$	−1 143.267456	6	5	1	0	−1 171.938843	1	2 449.775391

*In electron volts.
[a]Negative energy states, not observable.

and $l = L2 = -k$ (used with $k < 0$) for Dirac levels, both for positive and negative values of the parameter s in Dirac levels or for positive or negative values of the square root in Eq. (10) of the Schrödinger relativistic levels.

The Schrödinger levels are calculated in Table I with the plus sign inside the s in Eq. (10) in the $E1S(+)$ column and with the minus sign inside the s in the $E2S(-)$ column. The Dirac levels are calculated in Table II with the plus sign of s in Eq. (20) in the $E1D(+)$ column and with the minus sign of s in the $E2D(-)$ column. In describing Schrödinger levels, the notations $n = N$ = main quantum number, $l = L$ = angular quantum number, and $n' = M$ = radial quantum number are used. The nln' notation used in the equations is defined as the NLM level in Table I (one should point out that the levels with $l = 0,1,2,3,4,5,\ldots$ are also called $s,p,d,f,g,\ldots$ levels in the spectroscopic notations). Similarly, we will use the notation $n = N$ = main quantum number, $n' = M$ = radial quantum number, $k = K = a(j + \frac{1}{2})$ = Dirac k number, and $l = L$ = angular quantum number to describe the Dirac energy levels (a = plus or minus sign at k). The $nlkn'$ notation used in the equations is defined as the $NLKM$ level in Table II.

Table I shows that the $E2S(-)$ column [calculated for a minus sign inside Eq. (10) defining s variable] has very deep energy levels for hydrogen, lithium, sodium, potassium, rubidium, and cesium, with corresponding Z of 1, 3, 11, 19, 37, and 55, respectively. The very deep levels (we call them the DDLs, which stands for deep Dirac levels) are formed for each case of $M = L$ in Table I ($1s, 3p, 5d, 7f, \ldots$, if we use the spectroscopic notation). For the case of hydrogen, the DDLs have energies from 507 to 510 keV (see Table I.A). For higher Z atoms, values reach between 281 and 470 keV for cesium ($Z = 55$) (see Table I.F).

Similarly, Table II shows that the $E2D(-)$ column [calculated for a minus sign inside Eq. (20) defining s variable] indicates the existence of the DDLs in the case of the Dirac solution. The DDLs are formed for each case of $M = K$ in Table II ($2s, 4p, 6d, \ldots$, if we use the usual spectroscopic notation). The spectroscopic notation in Table II is given only for $NMKL1$ levels. In Table II.A for hydrogen, one can see the DDLs with energies from 509 to 510 keV. Similar results can be seen in Table II for lithium, sodium, potassium, rubidium, and cesium, with a corresponding Z of 3, 11, 19, 37, and 55, respectively. The DDLs are less deep with growing Z, reaching values between 406 and 477 keV for cesium ($Z = 55$) (see Table II.F).

If we compare the position of the DDLs in Table II with the DDLs in Table I, we can see that they are not in the same quantum number position. This difference is caused by the additional Dirac quantum number $k = K$, which is not present in the relativistic Schrödinger equation (only nln' quantum numbers are present in both theories). At present, we prefer to use the Dirac levels. Only the experiments could determine which theory is more correct.

The selection rules for transitions between $E1D(+)$ and $E2D(-)$ levels are not known. They would have to be determined experimentally. The neutralization of ions by electrons taking place on the electrodes during the the electrolysis could lead to a DDL formation.

It is well known that the Dirac equation allows solutions with the negative energy (see Ref. 1, p. 487). Also in our case, the $E2S(-)$ levels with $L > M$ in Tables I.A through I.F and the $E2D(-)$ levels with $K > M$ in Tables II.A through II.F correspond to negative masses and negative energy states. Such negative energy states cannot be observed as energy lev-

els in atoms with electrons because they correspond to transitions in the positron-antiproton atoms (antimatter atoms). They are denoted by footnote a in Tables I.A through I.F and II.A through II.F.

If the atomic transitions to the DDLs could be performed on earth, they would release an enormous energy, i.e., up to several mega-electron-volts per atom, if more than one electron per atom collapses on the DDL.

We suggest the following interesting experimental verification of the presented model. An atom (A, Z) with one electron bound on the DDL will have one of its nuclear charges screened by this electron (which is very close to the nucleus). This means that such an atom will chemically behave like element $(A, Z - 1)$, i.e., like a lighter element with $Z - 1$ electrons, which is its neighbor in the periodic table. For example, the alkali metal atoms with one electron on the DDL, created for instance in electrolysis, will behave like their lighter neighbors, noble gases, and should escape with the electrolytic gases. These noble gas masses could be determined by mass spectroscopy.

Results in Table I and II are valid for single-electron atoms only. More correct results can be obtained for many electron atoms by using the DHF calculation as presented in Ref. 6.

V. ATOMS FORMED FROM OTHER ELEMENTARY PARTICLES

It is well known experimentally that one can form atoms from e^+e^- (positronium), μ^--proton (muonium), and antiproton-proton (baryonium) pairs. It is theoretically possible to do the same with many other particles; i.e., one could form atomic pairs of $\mu^+\mu^-$, $\tau^+\tau^-$, $e^-\mu^+$, $e^-\tau^+$, $\mu^-\tau^+$, W^+W^-, quark-antiquark, etc. One could expect that these atomic pairs were present in the early universe. Our question is as follows: "Can these atomic structures be bound on the DDLs?"

Let us first consider the case of positronium. To calculate the positronium energy by the relativistic Schrödinger and Dirac equations, we again used Eqs. (6) and (24) with only one modification; we used a standard transformation to the center of mass system by replacing the electron mass with the reduced mass of the e^+e^- system. The results for the Dirac solution are shown in Table III.

We have calculated the normal energy levels as expected (the main transition of the positronium is the well-known 5.1-eV emission line, the transition between levels $2s$ and $1s$). In addition, we see that the positronium also has the DDLs (E_{DDL}) between 254.5 and 255.5 keV. Using a simple form of the Bohr model,[a] one can calculate that the 254.5 keV corresponds to a radius of ~2.85 fm, the γ is 1.992, and the v/c is ~0.865. It is interesting to note that the DDL orbit is reached when the reduced mass doubles, i.e., when the relativistic mass reaches the mass of one electron. Because the size of leptons is currently believed to be point-like on a scale of $<10^{-18}$ m, one expects that the electron and positron will not hit each other if they are on the stable DDL orbit. However, they are very close to each other and their wave functions overlap. The DDL transition mechanism could explain why the e^+e^- pair of the positronium could get close to each other and subsequently annihilate with some delay. The conventional application of the Heisenberg principle of uncertainty, $dE\,dt \geq \hbar/2$, would indicate a very narrow energy

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[a]Bohr model: $r_{min} = KZe^2/(2E_{DDL})$, where $Ke^2 = 14.4$ eV$\cdot A^0$, $Z = 1$; $\gamma = 1 + Ze^2/(2mc^2 r_{min}) = 1 + E_{DDL}/mc^2$, where mc^2 is reduced rest mass of the system; $v/c = (1 - 1/\gamma^2)^{1/2}$.

TABLE III

Dirac Energy Levels of the Positronium $(Z = 1)$*

	$E(N,Z)$	N	M	K	$L1$	E1D	$L2$	E2D
$1s$	-6.802913	1	0	1	0	-6.802930	1	-6.802930^a
$2p$	-1.700727	2	0	2	1	-1.700715	2	-1.700715^a
$2s$	-1.700727	2	1	1	0	-1.700738	1	$-254\,566.625000$
$3d$	-0.755879	3	0	3	2	-0.755872	3	-0.755872^a
$3p$	-0.755879	3	1	2	1	-0.755874	2	-6.802749^a
$3s$	-0.755879	3	2	1	0	-0.755881	1	-6.802217
$4f$	-0.425182	4	0	4	3	-0.425178	4	-0.425178^a
$4d$	-0.425182	4	1	3	2	-0.425178	3	-1.700708^a
$4p$	-0.425182	4	2	2	1	-0.425179	2	$-255\,032.000000$
$4s$	-0.425182	4	3	1	0	-0.425182	1	-1.700648
$5g$	-0.272116	5	0	5	4	-0.272114	5	-0.272114^a
$5f$	-0.272116	5	1	4	3	-0.272114	4	-0.755871^a
$5d$	-0.272116	5	2	3	2	-0.272114	3	-6.802697^a
$5p$	-0.272116	5	3	2	1	-0.272115	2	-6.802385
$5s$	-0.272116	5	4	1	0	-0.272116	1	-0.755855
$6h$	-0.188970	6	0	6	5	-0.188968	6	-0.188968^a
$6g$	-0.188970	6	1	5	4	-0.188968	5	-0.424178^a
$6f$	-0.188970	6	2	4	3	-0.188968	4	-1.700704^a
$6d$	-0.188970	6	3	3	2	-0.188968	3	$-255\,165.562000$
$6p$	-0.188970	6	4	2	1	-0.188968	2	-1.700669
$6s$	-0.188970	6	5	1	0	-0.188969	1	-0.425171

*In electron volts.
[a]Negative energy states, not observable.

resolution because the duration of the DDL transition is probably longer than a transition between normal atomic levels ($>10^{-9}$ s). However, when the electron and positron of the positronium atom fall to the DDL, the positronium atom could radiate either one photon of 253-keV energy or a large number of small photons, some of them in the ultraviolet (UV) range. This technical note does not provide the exact theory of transition to the DDL. However, if we are dealing with a multiphoton transition and not all photons are measured, the energy width of the DDL transition could be very broad, around or below 253 keV.

We are proposing therefore a new positronium experiment that would search for multiple photons with a hermetic detector arranged into a 4π geometry with a large segmentation of the photon detector, excellent energy resolution in both UV and X-ray range, and excellent timing characteristics. Measurements in Refs. 7 through 10 indicate so far that the final phase of the positronium annihilation proceeds mainly via (a) positronium in a ^{1}S-state $\rightarrow$ 2 photons (more than 95%) and (b) positronium in a ^{3}S-state $\rightarrow$ 3 photons. We would also like to suggest reanalyzing the positronium experiments performed so far to see if they are supporting the DDL mechanism already.

One can also apply the DDL theory to other leptons and quarks. For example, (a) for $\mu^+\mu^-$ atom, the lowest DDL energy level is 52.4 MeV, which would correspond to a Bohr radius of $r_{min} \approx 1.4 \times 10^{-2}$ fm, (b) for $\tau^+\tau^-$ atom, the lowest DDL energy level is 885.5 MeV and $r_{min} \approx 8.1 \times 10^{-4}$ fm, and (c) for $e^-\mu^+$ atom, the lowest DDL energy level is 504.8 keV and $r_{min} \approx 1.4$ fm, etc. For comparison, the Bohr DDL radius of the hydrogen atom is ~ 1.45 fm.[a] The DDL

theory could play a role in explaining the mechanism of the annihilation sequence.

If this model is proven correct by experiment, it would help to explain many outstanding problems such as dark matter in the universe. For instance, if a hydrogen atom can exist in the DDL state indefinitely, it would behave as a neutral particle that does not radiate and as dark matter. The electron capture by a proton in the hydrogen atom is possible under some energetic condition; if the electron is on the DDL, one needs only ~ 0.271 MeV to convert the hydrogen atom into a neutron. This energy could be obtained either from a gain in electrostatic energy when the DDL electron falls into the nucleus (below its stable orbit), or from a collision with a quark of the proton. Another possibility is to capture the electron fallen on the DDL, using a part of the energy from DDL transition for conversion of one proton in the nucleus to a neutron. If this mechanism is possible, the dark matter in the universe could be composed of hydrogen atoms with electrons on the DDL.

VI. CONCLUDING REMARKS

Definite conclusions could be made only after calculating the electron density of the electrons bound on the DDL (Ref. 11). However, some preliminary conclusions could be drawn from the presented results.

The theory presented here shows that the electrons bound on the E2S($-$) Schrödinger levels or the E2D($-$) Dirac levels should exist for all elements of the periodic system. The tables of electron levels can be similarly calculated as shown in Tables I and II for all elements and all quantum numbers n.

The result of electron-positron annihilation should be re-evaluated using the calculated DDLs for positronium. The DDL could also exist in the elementary particles that are forming "atoms" (two elementary particles bound by electrostatic forces). The DDL mechanism could help in understanding the annihilation sequence of particle-antiparticle.

The atoms with electrons on the DDL formed in the universe could have important applications in cosmology. If this model is proven correct experimentally, it could explain the outstanding problem of dark matter in the universe.

A single-electron atomic transition on the DDL should produce a very large energy release, in the region of 300 to 511 keV per transition. If such transitions could be made on the surface of the earth, it could produce an enormous amount of energy, which could be used practically.

The energy release in the DDL transitions could occur in a number of small quanta that together will give the binding energy of ~ 500 keV, similar to synchrotron radiation of an electron. This released energy could be practically observed as some form of heat energy.

An atom (A, Z) with one electron bound on the DDL will have one of its nuclear charges screened by this electron (which is very close to the nucleus). This means that such an atom will chemically behave like element $(A, Z - 1)$, i.e., like a lighter element with $Z - 1$ electrons, which is its neighbor in the periodic table. The alkali metal atoms with one electron on the DDL, created in electrolysis, will behave like their lighter neighbors, noble gases, and should escape with the electrolytic gases. These "alkali-noble gases" could be determined by mass spectrometry.

The recently observed excess heat release[12] in the electrolysis of LiOD in D_2O (see Refs. 13 and 14 and very recently the results in Ref.15) or the excess heat release observed in the electrolysis of K_2CO_3 and Rb_2CO_3 in H_2O (Refs. 16 and 17) could be caused by electron transitions on the DDLs. The lithium has DDLs with an energy -505 keV, and potassium and rubidium have -475 and -441 keV, respectively. Transitions of the electrons on those levels [and other observable $(-)$ levels shown in Tables I and II] could cause the excess heat release observed in the electrolysis of alkali metals.

The hydrogen or deuterium atoms with an electron bound on the DDL would behave almost like neutral particles with a Coulomb repulsion radius of 3 to 5 F (Ref. 11). They would penetrate rather freely through a crystalline lattice and through the electron shells of atoms forming such a lattice. Such atoms could then participate in nuclear reactions with hydrogen, deuterium, lithium, or palladium.

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